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TITLE – Analytical Evaluation of Launch Strategy Using Past Countdown Hold Data

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ABSTRACT

Using combinatorial analysis and past countdown data in the form of cumulative probabilities of total hold time in given countdown intervals, equations have been derived evaluating the probability of successful launch as a function of launch strategy. From these equations, the following conclusions can be drawn:

1. If the probability of a launch-on-time without built-in-holds (B.I.H's) is small, substantial improvement can be obtained by using a combination of B.I.H's as late in the countdown as possible, and a launch window.

2. There is little advantage in having a B.I.H. length exceeding the maximum value of total hold time found statistically for launches in the preceeding portion of the countdown, when unscheduled hold times greater than a certain maximum lead to a recycling (scrub) of the entire countdown.

3. Similarly, there is little advantage in employing a launch window greater than the peak value of the total hold time found statistically for the entire launch countdown.

4. There is little to be gained in employing a continuous launch window rather than two window panes, unless the window exceeds several hours.

As an illustration, applying Saturn I and IB countdown hold statistics to Saturn V L.O.R Missions we obtain:

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Abstract (contd.)

P(Launch-on-time without B.I.H's) = 0.13

For a combination of two hours of B.I.H at T = 22 minutes and a two hour launch window, and allowing for one recycle during the final 22 minutes of countdown, the probability of a successful launch was found to be:

P(L) = 0.68 with a continuous launch window

P(L) = 0.65 with two window panes.

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SUBJECT: Analytical Evaluation of Launch Strategy Using Past Countdown Hold Data

DATE: April 11, 1968

FROM: W. B. Gevarter

TM-68-2014-3

TECHNICAL MEMORANDUM

I. INTRODUCTION

This memorandum indicates how countdown data from past launches can be used to predict the probability of a successful launch, P(L), as a function of a chosen launch strategy. By launch strategy, we mean a particular arrangement of built-in-holds, launch window lengths, or window panes, and recycle requirements. The approach used in this memorandum is combinatorial analysis utilizing total hold time statistics from past launches. This approach is straightforward and very flexible.

Various authors have attacked this and related problems using a variety of methods. I have found it convenient to group these methods as follows:

1. Micromodeling of Event Successes, Failures, and Repairs, in the Order of the Countdown Schedule of Events

   This approach usually employs the traditional exponential failure rates utilized in standard reliability analyses. The success of equipments and events in the countdown are treated individually in the order of their occurrence. Examples of this approach are References 1, 2, 3, 4, and 5.

2. Macromodeling of Event Successes, Failures and Repairs

   These treat the entire countdown as a single unit, but the same methods are applied as in Group 1. Examples are References 6 and 7.

3. Macromodeling, Using Given Event Success Statistics

   In this approach, statistics on the success of all or portions of the countdown are utilized, instead of failure and repair rates, for the purpose of predicting the probability of launch for a given launch strategy. Examples of this are References 8, 9, and 10.

4. Probability of Success for Independent Trials

In addition to the probability of launch of a given vehicle, several authors have considered the probability of success of independent parallel events, such as multiple launchings. Examples of this are References 11, 12, and 13.

The approach used in this memorandum belongs with those in Group 3. The distinguishing feature is the handling of the statistical data (so as to facilitate the computations) and the particular launch strategies chosen for consideration.

II. THE UNSCHEDULED TOTAL HOLD TIME DISTRIBUTION

The key to efficient utilization of past data in predicting P(L) is to put the data in the most appropriate form. Reference 14 and most of the other references break the data up into occurrence of holds and individual hold times. For our purposes, it appears that the most expeditious way to order the data is by the total hold time in a countdown time interval.

Let:

${}_aH_b \triangleq$ total unscheduled hold time (for a given launch) present in the time interval $a \leq T < b$

where

$T \triangleq$ countdown script time* in minutes before launch

Then the cumulative probability distribution of ${}_aH_b$ is given by

$$F_{{}_aH_b}(h) = P({}_aH_b \leq h) \tag{1}$$

*Script time is the scheduled time, not considering holds, for the occurance of countdown events.

Using the data of Reference 14, Figure 1 presents this distribution (for launches*) for several time intervals for Saturn I and IB vehicles. The data satisfies reasonably well the constraint

$$F_{{}_0H_{T_T}}(h) = \int_0^{\infty}\int_0^{h} \frac{\partial F_{{}_0H_{22}}(x)}{\partial x} \; \frac{\partial F_{{}_{22}H_{T_T}}(h'-x)}{\partial x} \, dh'dx \qquad (2)$$

required for statistical independence of holds in the two intervals 0 to 22 minutes and 22 to T_T minutes.

III. PROBABILITY OF LAUNCH WITHIN A GIVEN CONTINUOUS LAUNCH WINDOW OF LENGTH, L.W.

In this and the next section we will derive the probability of launch for several launch strategies. Certain assumptions of independence of events will be apparent from the combinatorial analysis indicated. For simplicity we will consider secondary failures during a hold to be negligible. This latter assumption is warranted based on the data of Reference 14.

A. No B.I.H.'s or Recycling

For this simple case of no built-in holds (B.I.H.'s)

$$P(L) = P\{\text{total unscheduled hold time} \leq \text{launch window}\}$$

$$= F_{{}_0H_{T_T}}(\text{L.W.}) \qquad (3)$$

---

*Appendix C indicates how the statistics can be separated into those for launches and scrubs.

where

T_T = total countdown time (for the launch countdown)

L.W. = launch window width

B. Recycling but No B.I.H.'s

Certain launch vehicles (for reasons such as cryogenic or equipment limitations) require recycling if during the terminal portion of the countdown a hold exceeding a few minutes occurs (e.g. see Reference 15). For these cases, if a hold occurs at a countdown script time of T_h during the terminal period, the extra time required by the recycle acts to further reduce the launch window remaining after the holds earlier in the countdown have been subtracted from it. Thus, if recycling to T_c is required for unscheduled holds at $T<T_c$, then (considering only one recycle):

$$P(L) = P\Big\{\left[{}_{T_c}H_{T_T} \le \text{L.W. and } {}_0H_{T_c} = 0\right]$$

$$\text{or } \left[{}_{T_c}H_{T_T} < \text{L.W. and } 0 < {}_0H_{T_c} < \left(\text{L.W.} - {}_{T_c}H_{T_T} - h_r\right) \text{ and } {}_0^rH_{T_c} = 0\right]\Big\} \tag{4}$$

$$= F_{{}_{T_c}H_{T_T}}(\text{L.W.})\, F_{{}_0H_{T_c}}(0)$$

$$+ \int_0^{\text{L.W.}} \left[\Delta F_{{}_0H_{T_c}}(\text{L.W.}-h-h_r)\right] f_{{}_{T_c}H_{T_T}}(h)\,dh\; {}^rF_{{}_0H_{T_c}}(0)$$

where

$$h_r = \text{time lost due to recycling.} \tag{5}$$
$$\approx T_c - T_h$$

$$f(h) = \frac{d}{dh} F(h) \tag{6}$$

${}^{r}F_{{}_0H_{T_c}}(h)$ = hold distribution during the terminal period after recycling has occurred.

$$\Delta F_{{}_0H_{T_c}}(h) \triangleq \begin{cases} F_{{}_0H_{T_c}}(h) - F_{{}_0H_{T_c}}(0), \text{ if } h>0 \\ 0, \text{ otherwise.} \end{cases} \tag{7}$$

C. <u>B.I.H. at $T = T_b$ but No Recycling Required</u>

If we have a B.I.H. at $T = T_b$, it acts like a launch window by absorbing holds occurring earlier in the countdown. However, it differs from a launch window in that the vehicle must wait thru any of the B.I.H. time, h_b, remaining before continuing the countdown.

In addition to absorbing holds earlier in the countdown, B.I.H.'s act as an umbrella during which incipient problems that would cause holds later in the countdown can be corrected. Thus, a B.I.H. may also serve to reduce the probability of holds later in the countdown. This reduction is attributed to the possible correction of discrepancies not in the critical path that are apparent at T_b, but for which ordinarily no hold would be called until later in the countdown. The normal delay in the calling of the hold for this type of discrepancy is due to the following factors:

1. Holds should be called as late in the countdown as possible, so that more than one discrepancy can be attended to in the same hold.

2. Critical path discrepancies might incur a hold during which the latent discrepancy might be repaired without a separate hold.

3. The discrepancy might correct itself.

Based on the preceding discussion, the P(L) for the case of a B.I.H. at $T = T_b$, of duration h_b, is given by

$$P(L) = P \Big\{ \Big[{}_{T_b}H_{T_T} \leq h_b \text{ and } {}^{b}_{0}H_{T_b} \leq L.W. \Big]$$

$$\text{or} \left[h_b < {}_{T_b}H_{T_T} \leq \left(h_b + L.W.\right) \text{ and } {}_0H_{T_b} \leq \left(L.W. + h_b - {}_{T_b}H_{T_T}\right)\right]\right\}$$

$$= F_{{}_{T_b}H_{T_T}}(h_b)\ {}^bF_{{}_0H_{T_b}}(L.W.) + \int_{h_b}^{h_b + L.W.} f_{{}_{T_b}H_{T_T}}(h)\ F_{{}_0H_{T_b}}(L.W. + h_b - h)dh \tag{8}$$

where ${}^bF_{{}_0H_{T_b}}(h)$ is the hold time distribution for the count-down interval 0 to T_b, as modified by the effects of the B.I.H.

D. <u>B.I.H. at $T = T_c$ and Recycling</u>

Similarly (following the arguments in Sections B and C) for a B.I.H. at $T = T_c$, and the requirement that for holds at $T<T_c$ we recycle to $T = T_c$ (allowing only one recycle), we have:

$$P(L) = P\left[{}_{T_c}H_{T_T} \leq (L.W. + h_b) \text{ and } {}^b_0H_{T_c} = 0\right]$$

$$\text{or } \left[{}_{T_c}H_{T_T} \leq h_b \text{ and } 0 < {}^b_0H_{T_c} \leq \left(L.W. - h_r\right) \text{ and } {}^r_0H_{T_c} = 0\right] \tag{9}$$

$$\text{or } \left[h_b < {}_{T_c}H_{T_T} \leq \left(L.W. + h_b\right) \text{ and } 0 < {}_0H_{T_c} \leq \left(L.W. + h_b - h_r - {}_{T_c}H_{T_T}\right) \text{ and } {}^r_0H_{T_c} = 0\right]\right\}$$

$$= F_{{}_{T_c}H_{T_T}}(L.W. + h_b)\ {}^bF_{{}_0H_{T_c}}(0)$$

$$+ F_{{}_{T_c}H_{T_T}}(h_b) \left[\Delta^b F_{{}_0H_{T_c}}(L.W.-h_r)\right] {}^rF_{{}_0H_{T_c}}(0)$$

$$+\int_{-h_b}^{L.W.+h_b} f_{{}_{T_c}H_{T_T}}(h) \left[\Delta F_{{}_0H_{T_c}}(L.W. + h_b-h_r-h)\right]dh \; {}^rF_{{}_0H_{T_c}}(0)$$

IV. P(L) FOR A GIVEN LAUNCH WINDOW CONSISTING OF TWO PANES (OF NEGLIGIBLE WIDTH) L.W. APART

To save software, training, analysis, and launch system complexity, it is often advantageous to limit launches to several points within the launch window rather than to plan for a continuous launch window. The simplest and most extreme version of this is to have a small launch pane at the beginning and end of the launch window. It is this extreme that we will consider here.

A. No B.I.H.'s or Recycling

For this case, we plan to nominally launch at the first pane, so that if a hold occurs we proceed to T = 0 and hold until the second pane occurs. Therefore, if we miss the first pane, this is equivalent to having a B.I.H. at T = 0 of length = L.W. Thus

$$\begin{aligned} P(L) &= P\left\{{}_0H_{T_T} \leq L.W.\right\} \\ &= F_{{}_0H_{T_T}}(L.W.) \end{aligned} \tag{10}$$

the same results as Equation (3).

B. Recycling but No B.I.H.'s

If recycling to T_c is required for holds at $T<T_c$, only one recycle being allowed, the launch strategy is that if holds occur at $T \geq T_c$, we hold at T_c until it is time to try for the second window pane. For this case

$$P(L) = P\left\{\left[{}_{T_c}H_{T_T} = 0 \text{ and } {}_0H_{T_c} = 0\right]\right.$$

$$\text{or} \quad \left[0 < {}_{T_c}H_{T_T} \leq (L.W.) \text{ and } {}^B_0H_{T_c} = 0\right]$$

$$\text{or} \quad \left[{}_{T_c}H_{T_T} = 0 \text{ and } 0 < {}_0H_{T_c} \leq \left(L.W.-h_r \right) \text{ and } {}_0^rH_{T_c} = 0 \right]$$

$$= F_{{}_{T_c}H_{T_T}}(0)\, F_{{}_0H_{T_c}}(0) + \Delta F_{{}_{T_c}H_{T_T}}(L.W.)\, {}^BF_{{}_0H_{T_c}}(0) \Big\} \quad (11)$$

$$+ F_{{}_{T_c}H_{T_T}}(0)\, \Delta F_{{}_0H_{T_c}}(L.W.-h_r)\, {}^rF_{{}_0H_{T_c}}(0)$$

where

${}^BF_{{}_0H_{T_c}}(h) \triangleq$ the hold time distribution in the interval 0 to T_c after waiting for the second pane

C. B.I.H. at $T = T_b$ but No Recycling Required

This is similar to a continuous launch window with a B.I.H. except for the modification considered in Case A. Thus from Equation (8):

$$P(L) = P\left\{ \left[{}_{T_b}H_{T_T} \leq h_b \text{ and } {}_0^bH_{T_b} \leq L.W. \right] \right.$$

$$\left. \text{or} \quad \left[h_b < {}_{T_b}H_{T_T} \leq \left(h_b + L.W. \right) \text{ and } {}_0H_{T_b} \leq \left(L.W. + h_b - {}_{T_b}H_{T_T} \right) \right] \right\}$$

$$= F_{{}_{T_b}H_{T_T}}(h_b)\, {}^bF_{{}_0H_{T_b}}(L.W.)$$

$$+ \int_{h_b}^{h_b+L.W.} f_{{}_{T_b}H_{T_T}}(h)\, F_{{}_0H_{T_b}}(L.W. + h_b - h)\, dh \quad (12)$$

D. B.I.H. at $T = T_c$ and Recycling

For a B.I.H. at $T = T_c$, the recycle point, we hold at $T = T_c$ until time for the first or second pane. If the first pane is available and unscheduled holds after $T = T_c$ occur, we recycle to $T = T_c$ and hold for the second pane. Thus,

$$P(L) = P\Big\{\Big[{}_{T_c}H_{T_T} \le h_b \text{ and } {}_0^bH_{T_c} = 0\Big]$$

$$\text{or} \quad \Big[h_b < {}_{T_c}H_{T_T} \le \big(h_b + L.W.\big) \text{ and } {}_0^BH_{T_c} = 0\Big]$$

$$\text{or} \quad \Big[{}_{T_c}H_{T_T} \le h_b \text{ and } 0 < {}_0^bH_{T_c} \le \big(L.W. - h_r\big)$$

$$\text{and } {}_0^rH_{T_c} = 0\Big]\Big\}$$

$$= F_{{}_{T_c}H_{T_T}}(h_b)\ {}^bF_{{}_0H_{T_c}}(0)$$

$$+ {}_b\Delta F_{{}_{T_c}H_{T_T}}(h_b + L.W.)\ {}^BF_{{}_0H_{T_c}}(0)$$

$$+ F_{{}_{T_c}H_{T_T}}(h_b)\ \Delta\,{}^bF_{{}_0H_{T_c}}(L.W. - h_r)\,{}^rF_{{}_0H_{T_c}}(0) \qquad (13)$$

where

$$ {}_b\Delta F_{{}_{T_c}H_{T_T}}(h_b + L.W.) = \begin{cases} F_{{}_{T_c}H_{T_T}}(L.W. + h_b) - F_{{}_{T_c}H_{T_T}}(h_b) & L.W. > 0 \\ 0 & L.W. \le 0 \end{cases} \qquad (14)$$

It will be observed that due to the assumption of no secondary failures, that the results with panes are the same as for a continuous window, except for cases involving recycling.

V. OTHER LAUNCH STRATEGIES

The combinatorial analysis approach of Sections III and IV can be extended in a straight forward manner to include other launch strategies, consisting of any combination of B.I.H.'s, launch window widths, and number of panes.

VI. APPLICATION TO SATURN V L.O.R. MISSION

We will illustrate the use of the formulas by applying them to Saturn V L.O.R. countdowns. For the purposes of this example we will use the simple linear approximations to the countdown hold statistics from Saturn I and IB launches given in Figure 1. Ideally, it would be worthwhile to modify the hold statistics to account for the differences in vehicle complexities and the points in the countdown at which comparable events occur. However, the general trends of the hold statistics for various vehicles, studied in Reference 14, were sufficiently similar, that it appears reasonable for a first approximation to apply these statistics without modifications, except those resulting from B.I.H.'s and recycling.

Appendices A and B provide a simple, first-order method for modifying the hold statistics to account for the effect of B.I.H.'s and recycling.

Using this approach, and assumed values for the parameters of

$$k_r = .5$$

$$k_b' = .7$$

$$T_b = T_c = 22 \text{ mins.}$$

$$h_r = 20 \text{ mins.}$$

the total hold statistics, based on a linearization of Saturn I and IB statistics, have been plotted in Figures 2 and 3 in a form suitable for the estimation.

Reference (15) indicates that the closest to launch that an extended built-in-hold can be incorporated in the Saturn V is at a countdown time of 22 minutes before launch. If a hold longer than a few minutes occurs in the remaining portion of the countdown, then, because of cryogenic problems, the vehicle must recycle to T = 22 minutes. Thus a countdown time of 22 minutes has been chosen as the recycle point, and the time for the B.I.H. in our example.

Using Equations (9) and (C1), and the statistics of Figures 2 and 3, a plot of the P(L) for Saturn V for a continuous launch window, as a function of the length of built-in-holds at T_b = 22 minutes, is given in Figure 4. Similarly, using Equation (13), the P(L) plot shown in Figure 5 is obtained for the case of a discontinuous launch window consisting of two small window panes a time interval L.W. apart. From the figures we observe that substantial improvements in P(L) result from incorporating a built-in-hold and launch window of about two hours duration for each. Also, when the hold time statistics for launches and scrubs are well separated, as in our example, there appears to be little advantage in having a built-in-hold exceeding the peak value of the total hold time (for no scrub) found statistically in the preceding portion of the countdown. Similarly, there appears to be little advantage in having a launch window greater than the peak value of the no scrub total hold time found statistically for the entire countdown. Comparing Figures 4 and 5, the indication is that there is little to be gained in employing a continuous launch window rather than two window panes unless the launch window exceeds several hours.

For a combination of two hours of B.I.H. at T_b = 22 minutes and a two hour launch window, Figures 4 and 5 indicate that:

P(L) = 0.68 with a continuous window
P(L) = 0.65 with two window panes

versus the value of

P(L)=0.13 for launch-on-time and no B.I.H.'s.

VII. SUMMARY AND CONCLUSIONS

Using combinatorial analysis and past countdown data in the form of cumulative probabilities of total hold time in given countdown intervals, equations have been derived evaluating the probability of successful launch as a function of launch strategy. From these equations, the following conclusions, can be drawn:

1. If the probability of a launch-on-time without B.I.H's is small, substantial improvement can be obtained by using a combination of B.I.H.'s as late in the countdown as possible, and a launch window.

2. There is little advantage in having a B.I.H. length exceeding the maximum value of the total hold time found statistically for launches in the preceeding portion of the countdown, when unscheduled hold times greater than a certain maximum lead to a recycling (scrub) of the entire countdown.

3. Similarly, there is little advantage in employing a launch window greater than the peak value of the total hold time found statistically for the entire launch countdown.

4. There is little to be gained in employing a continuous launch window rather than two window panes unless the launch window exceeds several hours.

Applying Saturn I and IB countdown hold statistics to Saturn V L.O.R. Missions we obtain:

P(Launch-on-time without B.I.H.'s) = 0.13

For a combination of two hours of B.I.H. at T = 22 minutes and a two hour launch window, and allowing for one recycle during the final 22 minutes of countdown:

P(L) = 0.68 with a continuous launch window

P(L) = 0.65 with two window panes

Note that the small difference between a continuous window and panes is due in large measure to the assumption that there are no secondary failures during holds. However, as indicated by Ref. 14, this appears to be a reasonable assumption. The small rate of secondary failures that one could expect would serve to somewhat decrease the effectiveness of panes relative to a continuous window. It would also serve to somewhat reduce, from the values given in this memorandum, the P(L) as the launch window length or B.I.H. time is increased.

W. B. Gevarter

2014-WBG-bjh W. B. Gevarter

Attachments

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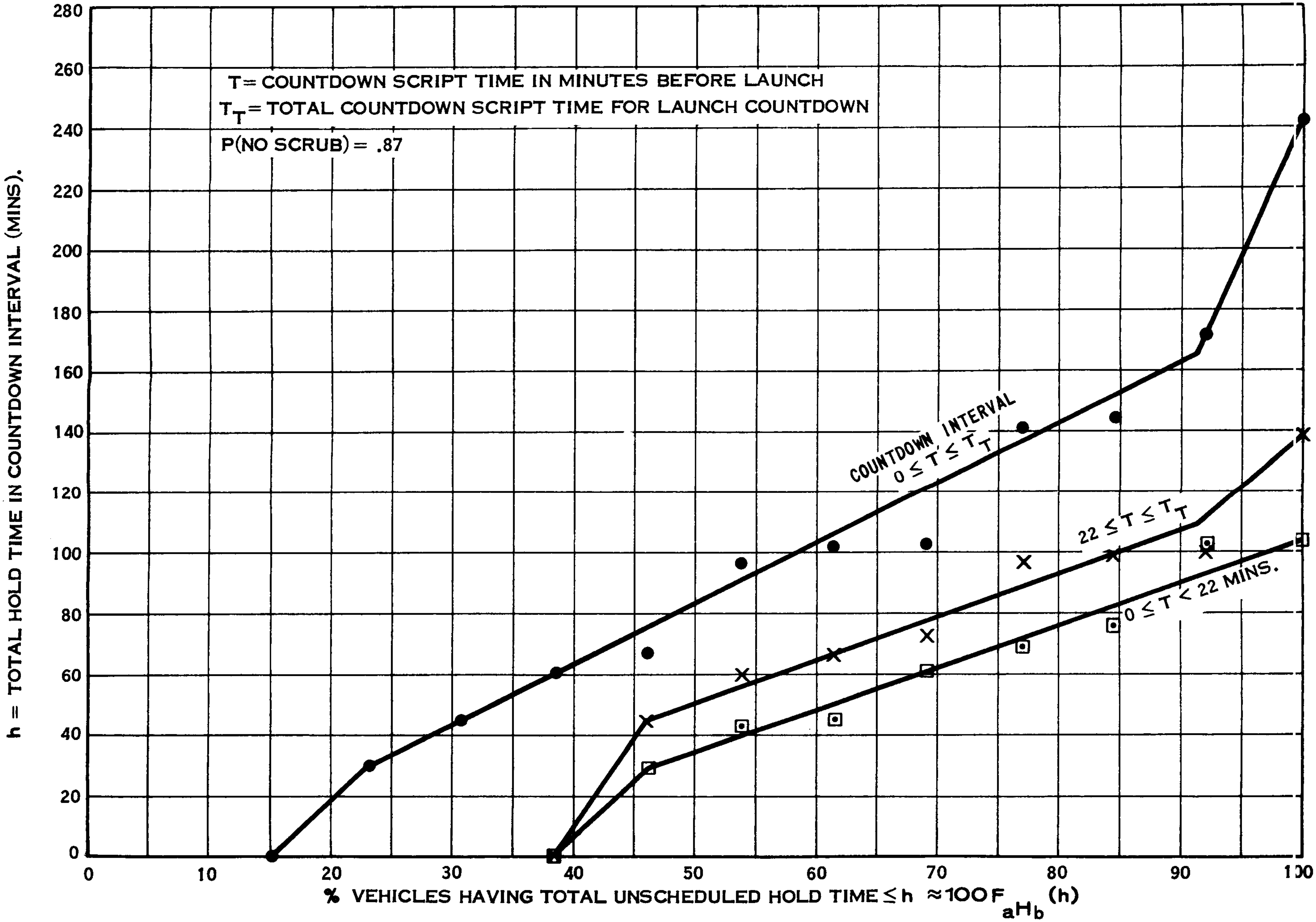

FIGURE 1. DISTRIBUTIONS OF TOTAL UNSCHEDULED HOLD TIMES, SATURN I AND IB. LAUNCHES ONLY

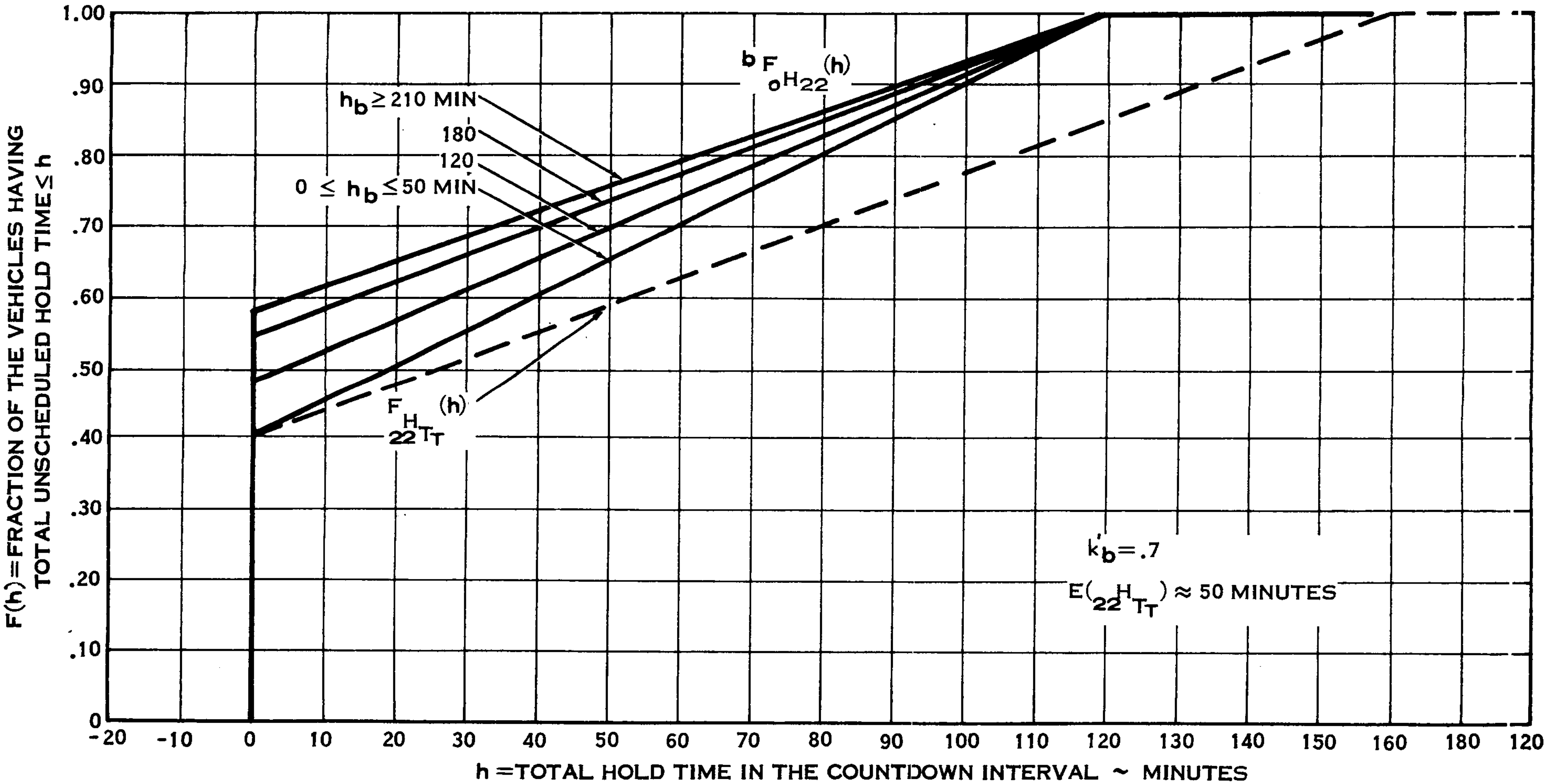

FIGURE 2. DISTRIBUTION OF INTERVAL TOTAL UNSCHEDULED HOLD TIMES WITH A B.I.H. AT T=22 MINUTES, LAUNCHES ONLY

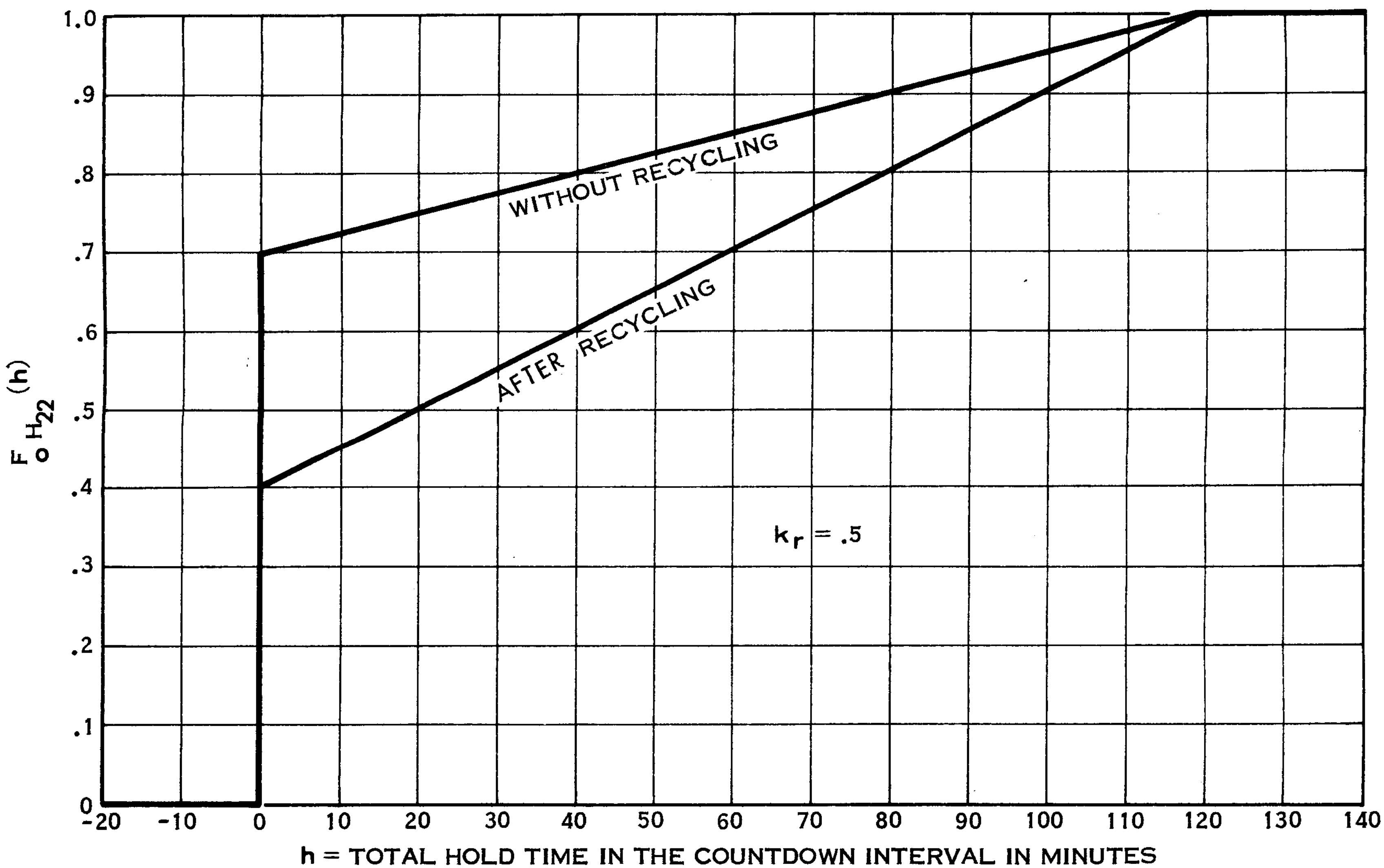

FIGURE 3. DISTRIBUTION OF TOTAL UNSCHEDULED HOLD TIME WITH AND WITHOUT RECYCLING IN THE COUNTDOWN INTERVAL 0 TO 22 MINUTES

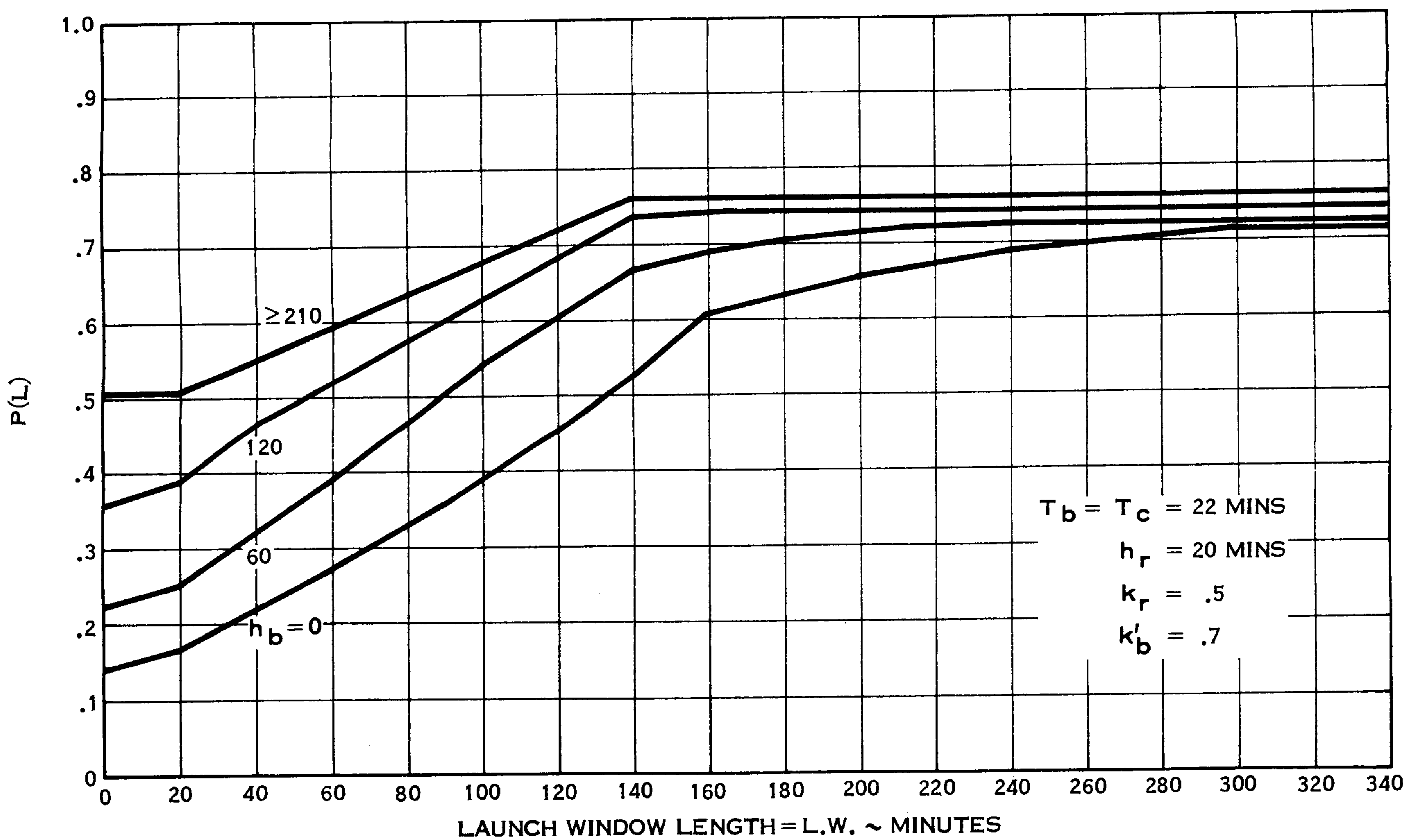

FIGURE 4. PROBABILITY OF A SATURN V L.O.R. LAUNCH FOR CONTINUOUS LAUNCH WINDOWS, USING SATURN I AND IB STATISTICS AND ALLOWING ONE RECYCLE (EQ. 9)

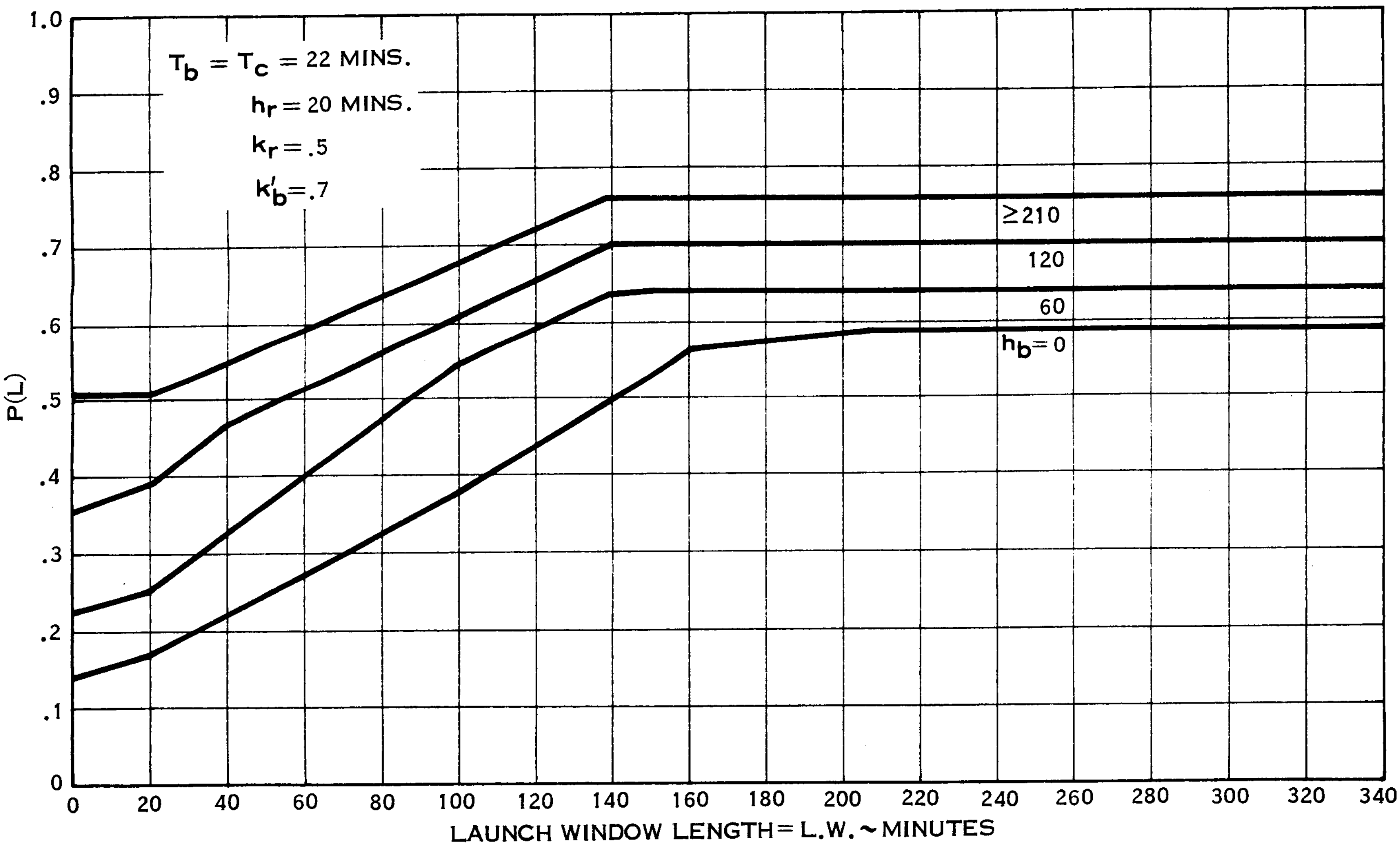

FIGURE 5. PROBABILITY OF SATURN V L.O.R. LAUNCH FOR TWO LAUNCH WINDOW PANES, L.W. APART, USING SATURN I AND IB STATISTICS AND ALLOWING FOR ONE RECYCLE (EQ. 13)

APPENDIX A

Derivation of the Effect of Recycling on the Probability Distribution for Total Holds Times

Reference 16 indicates that the Thor-Delta Vehicle recycles to T-8 if a extended hold occurs during the last 8 minutes of countdown. The data of Reference 16 tends to indicate that there was a reduction in the expected hold rate during the final 8 minutes of countdown (after a recycle) to about 1/2 of the expected hold rate before recycling. This might be attributed to the discovery and correction of discrepancies during the first try.

To account for this effect, we might modify F(h) so that the probability of exceeding a total hold time of length h after recycling is only k_r that of the probability before recycling. Thus

$$ {}^{r}P(H>h) = k_r\ P(H>h) = k_r\left[1-F(h)\right] = 1-{}^{r}F(h) \qquad \text{(A1)} $$

Solving Equation (A1) yields:

$$ {}^{r}F(h) = \begin{cases} k_r F(h) + \ 1-k_r & \text{if} > 0 \\ 0 & \text{if} \leq 0 \end{cases} \qquad \text{(A2)} $$

APPENDIX B

Derivation of the New Probability Distribution for Total Hold Times After a Built-In-Hold

There has been some viewpoints that indicate that not only will a B.I.H. of length h_b at countdown time, T_b, absorb all the total hold time of length h_b or less occurring earlier in the countdown, but it will also reduce the probability of holds later in the countdown. That is, such a hold has an umbrella effect during which potential and latent problems are corrected.

To account for this effect, we might modify F(h) so that the probability, of exceeding a total hold time of length h in the remaining countdown, is only k_b that of the probability without a B.I.H. Thus

$$\begin{aligned} {}^{b}P(H>h) &= k_b P(H>h) \\ &= k_b\left[1-F(h)\right] = 1-{}^{b}F(h) \end{aligned} \qquad \text{(B1)}$$

Solving Equation (B1) yields:

$${}^{b}F(h) = \begin{cases} \left(1-k_b\right) + k_b F(h) & \text{if } > 0 \\ 0 & \text{if } \leq 0 \end{cases} \qquad \text{(B2)}$$

To accommodate the effect of the length of the built-in-hold, we could use as a first approximation:

$$k_b = 1-m_b\left(1-k_b'\right) \qquad \text{(B3)}$$

where:

$$m_b \triangleq \left\{ \begin{array}{ll} \dfrac{h_b-E(h)}{h_{max}} \text{ for } 0 < & \dfrac{h_b-E(h)}{h_{max}} \leq 1 \\ \\ 1 \quad \text{for} & \dfrac{h_b-E(h)}{h_{max}} > 1 \end{array} \right\}$$

and h_{max} is the maximum value of h for which no scrubs were called. Observe that we have subtracted off the E(h), as the effect of this amount of hold has already been naturally included in the statistics.

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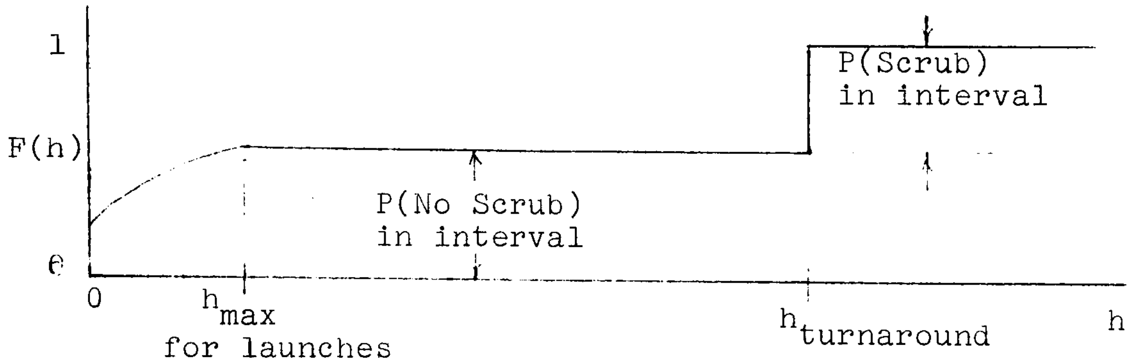
APPENDIX C

Separation of Hold Statistics into P(No Scrub) and the Cumulative Total Hold Time Probability Distribution for Launches

For some vehicles, either repairs can be made within a few hours, or the repair times are so long that a recycling (scrub) of the entire launch countdown is required. For these cases where the hold times for launches and scrubs are well separated, the accompanying sketch indicates a typical cumulative total hold time probability distribution in a countdown interval.



From the sketch it is obvious that for $h < h_{turnaround}$, we can represent the distribution in the factored form.

$$F(h) = P(\text{No Scrub})\, F_L(h) \qquad (C1)$$

where $F_L(h)$ = cumulative total hold time probability distribution, in the interval, for launches.